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POLYTECHNIC INST OF NEW YORK BROOKLYN DEPT OF MECHAN--ETC F/G 13/13  
STATIC AND DYNAMIC BEHAVIOR OF NONCIRCULAR CYLINDRICAL SHELLS.(U)  
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F44620-74-C-0047

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POLY-M/AE-77-5

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AFOSR - TR - 77 - 0212

Contract No. F44620-74-C-0047  
Project No. 9782-04

ADA 038203

FINAL REPORT

STATIC AND DYNAMIC BEHAVIOR OF  
NONCIRCULAR CYLINDRICAL SHELLS

by

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Department of  
Mechanical and Aerospace Engineering

February 1977

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POLY-M/AE Report No. 77-5

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REPORT DOCUMENTATION PAGE		READ INSTRUCTIONS BEFORE COMPLETING FORM	
1. REPORT NUMBER AFOSR-TR-77-0212	2. GOVT ACCESSION NO.	3. RECIPIENT'S CATALOG NUMBER repl	
4. TITLE (and Subtitle) STATIC AND DYNAMIC BEHAVIOR OF NONCIRCULAR CYLINDRICAL SHELLS	5. TYPE OF REPORT & PERIOD COVERED FINAL 1 Jan 74-31 Dec 76		
7. AUTHOR(s) JOSEPH KEMPNER	6. PERFORMING ORG. REPORT NUMBER POLY-M/AE Report No. 77-5		
	8. CONTRACT OR GRANT NUMBER(s) F44620-74-C-0047		
9. PERFORMING ORGANIZATION NAME AND ADDRESS POLYTECHNIC INSTITUTE OF NEW YORK DEPT OF MECHANICAL AND AEROSPACE ENGINEERING 333 JAY STREET, BROOKLYN, NEW YORK 11201		10. PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS 9782-04 61102F	
11. CONTROLLING OFFICE NAME AND ADDRESS AIR FORCE OFFICE OF SCIENTIFIC RESEARCH/NA BUILDING 410 BOLLING AIR FORCE BASE, D C 20332		12. REPORT DATE Feb 1977	
14. MONITORING AGENCY NAME & ADDRESS (if different from Controlling Office)		13. SECURITY CLASS. (of this report) UNCLASSIFIED	
12 23p.		13a. DECLASSIFICATION/DOWNGRADING SCHEDULE	
16. DISTRIBUTION STATEMENT (of this Report) Approved for public release; distribution is unlimited.			
16 9782 17 04			
17. DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if different from Report) 14 POLY-M/AE-77-5			
18. SUPPLEMENTARY NOTES			
19. KEY WORDS (Continue on reverse side if necessary and identify by block number) OVAL CYLINDERS BUCKLING POSTBUCKLING VIBRATIONS REINFORCED SHELLS			
20. ABSTRACT (Continue on reverse side if necessary and identify by block number) During the past contract, emphasis has been placed upon problems concerned with the buckling, postbuckling, and vibrations of rings and cylindrical shells of variable curvature. Some work was also performed on reinforced spherical and noncircular cylindrical shells. The list of references at the end of this report represents reports, publications, talks, and theses prepared during the course of the contract period.			

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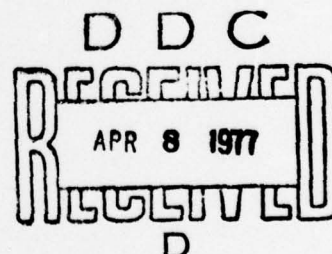


### Introduction

This report on the past contract period (January 1974 through December 1976) presents but a small part of the overall research effort carried out under the sponsorship of the Air Force covering a period of almost twenty years. A complete listing of the related reports and publications can be found in an appendix to the present report.

During the past contract, emphasis has been placed upon problems concerned with the buckling, postbuckling, and vibrations of rings and cylindrical shells of variable curvature. Some work was also performed on reinforced spherical and noncircular cylindrical shells. The list of references at the end of this report represents reports, publications, talks, and theses prepared during the course of the contract period. The writer would like to take this opportunity to thank those authors whose names appear on this list, particularly, Dr. Y. N. Chen as well as Professor B. Erickson, for their invaluable contributions to the work performed. He would also like to acknowledge the financial assistance provided by the AFOSR throughout the course of the studies summarized herein.

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Buckling and Postbuckling of Oval Cylindrical Shells (1, 8, 10)\*:

Buckling and initial postbuckling of an oval cylindrical shell under pure bending and under combined uniform axial compression and bending was investigated in order to ascertain the behavior of such a shell under combined loads (1). The first and second order stability equations were developed from the Donnell-type equations, which were shown to be appropriate. The solution of these two sets of equations were used to determine, respectively, the buckling characteristics and a "sensitivity parameter". The buckling loads were found to be in good agreement with the engineering approximation based upon the assumption that buckling occurs when the local axial stress equals that corresponding to the classical buckling stress of a locally equivalent circular cylindrical shell under uniform axial compression. Results also showed that an oval cylinder can be stronger or weaker than the equivalent circular cylinder depending upon the orientation of the couples. However, in any case the oval shell was found to be quite sensitive to imperfections; the greater the load carrying capacity, the greater the sensitivity. Furthermore, in contrast to the behavior of the circular and weak oval cylinders, it was found that buckling of the strong oval cylinder need not initiate at the position of maximum compressive stress.

This research was also extended to bifurcation type of buckling analysis of oval cylinders under the combined action of uniform compressive end thrust and terminal bending couples with arbitrary orientation (8). In this work the classical stability problem was formulated and solved. Computation of the critical interacting loads was performed for a series of orientation angles  $\beta$ ; viz.,  $\beta = 0^\circ, 2^\circ, 10^\circ, 15^\circ, 30^\circ, 45^\circ$ , and  $90^\circ$ , where  $\beta$  represents the angle the bending moment vector makes with the minor axis of the oval cross section. All calculations cover the entire range of the oval eccentricity parameter, which corresponds to the major-minor axes ratio in the range of 1.0 to 2.06. Both symmetric and antisymmetric modes of deformation were needed, and they were taken into account. Weak coupling between the two types of modes was found to exist. The numerical results obtained were analyzed and, as expected, the orientation angle  $\beta$  was found to have an important influence on the buckling loads, inasmuch as  $\beta = 0^\circ$  corresponds to the strong bending mode, while  $\beta = 90^\circ$  corresponds to the weak mode. Moreover, it was

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\* Numbers in brackets refer to listings under the section in Reports and Publications.



shown that an interpretation of the results can be viewed in the light of a practical engineering point of view. Essentially, the total axial compressive stress of an oval shell, at the point where buckles initiate, is found to be close to that of the linear buckling stress of an axially compressed circular cylinder whose thickness-to-radius ratio equals the corresponding local value at the buckling points of the oval. Such a simple engineering model for the estimation of the characteristic parameters, which was found applicable to simpler loading conditions in earlier studies in this program, once again proved to be justifiable in the general case of oval shells subjected to nonsymmetric, nonuniform loading.

Reinforced Spherical and Oval Shells (2, 3):

In the spherical shell investigation, studies of the state of stress in a shallow spherical shell containing an asymmetrically located stiffened circular hole subjected to an arbitrarily located concentrated load were performed with two aspects in mind (2, 3). The first was the investigation of the effect of a stiffening ring on the stresses in the vicinity of the hole when the shell is undergoing severe deformation due to an eccentrically applied concentrated load. In the second, the interaction of the hole with the outer boundary was studied for cases where the hole is located near the outer edge. For this case, the hole is placed close enough to the outer edge so that the perturbed stress field due to the hole is not fully decayed before it reaches the outer boundary. Consequently, there is an interaction between the two boundaries. In treating this problem, the boundary conditions were satisfied exactly at the hole boundary while the outer edge conditions were satisfied using the least square point matching method. Numerical results were presented and showed the effect of varying the ring stiffness parameters, load location, and hole location.

In the area of research involved with buckling of reinforced noncircular cylindrical shells compressed by axial forces in the presence of internal or external pressure some results have been found. The reinforcements considered includes rings and/or stringers placed either inside or outside of the shell. The shell in question is finite in length and, thus, the influence of various types of end supports was investigated. To date, a parametric study has almost been accomplished. The work includes the cases of reinforced shells, with or without lateral



pressure, for the types of simple supports known as S2 and S4, the clamped supports of classes C1 and C4, and the cases of reinforced shells with similar edge conditions but without lateral pressure.

#### Vibration of Oval Rings (4, 5, 6):

As a prelude to studies of the vibration characteristics of noncircular cylindrical shells, research was performed on the free and forced vibration of oval rings vibrating in their plane of curvature. This work determined the response of oval rings of varying eccentricity for several types of applied loading conditions. As anticipated, it was found that the magnitude as well as the frequency of oscillation of the deflection is dependent upon the ring eccentricity. In addition, the dependence of the deflection upon circumferential position was shown to be related to the local radius of curvature. Other results included the determination of the resonance frequencies of the ring as well as the existence of a double resonance "peak" observed via the classic "beat" phenomenon.

#### Vibrations of Oval Cylindrical Shells (7, 9, 10):

The free vibration of an oval cylindrical shell of finite length was investigated with the aid of the kinematic relations of the first-order shell theory of Sanders (7). These relations are readily reducible to those of the Donnell-type of shell theory via a tracing constant  $k_s$ . In-plane inertia was retained throughout the analysis. A method incorporating a type of eigenfunction expansion into Hamilton's principle, suitable for the present class of problems, was developed, and judged to be far more convenient than a parallel Fourier analysis.

In addition to the determination of the natural frequencies and deformation characteristics, attention was focused on the influence of various types of simple support and clamped conditions enforced at the edges of the shell. Two modes of deformation, corresponding to a "higher" and a "lower" frequency were observed to exist for every pair of axial and circumferential wave numbers, depending upon the degree of circumferential symmetry in the deformed pattern.

These studies have also been extended to deal with (but not restricted to) the free vibration problem of unsupported noncircular cylinders (9). The required modification included the enforcement of the edge conditions at the free ends of a finite cylindrical shell which are unsatisfied by the modal functions owing to the presence of the variable curvature terms. Such conditions were posed as additional constraints by way of the well-known formalism of Lagrange multipliers. In addition, it was found that the inclusion of the lowest modes, commonly approximated by the Rayleigh-Love modes, was essential to the completeness of the eigen-function representation. The validity of the proposed procedure of analysis was illustrated by its application to the solution of the free vibration problem of oval cylindrical shells with free ends.

With regard to related experimental studies, because of the complex behavior of the variable curvature cylinder undergoing vibrations, difficulties developed in the procedures for detecting high frequency displacement patterns. The attempt to use a speckle pattern interferometric method to observe displacements in real time and to obtain photographs of the wave shapes in this manner has turned out to be more difficult than was anticipated. A moiré method of the detection and observation is now being adapted, and it is hoped that this technique will serve to accomplish the desired determination of vibration characteristics of oval cylindrical shells.

Reports and Publications (January 1974 - December 1976):

1. Kempner, Joseph, and Chen, Youl-Nan: Buckling and Initial Postbuckling of Oval Cylindrical Shells Under Combined Axial Compression and Bending. Transactions of the New York Academy of Sciences, Series E, Vol. 36, No. 11, pp. 171-191, February 1974; also, authors each awarded the I. B. Laskowitz Gold Medal for "Research in Aerospace Engineering Sciences, Support Systems, and Components" by the New York Academy of Sciences, at the Academy's Annual Meeting on December 6, 1973 held at the Museum of Natural History.
2. Pifko, Allan B.: Analysis of a Shallow Spherical Shell Containing an Asymmetrically Located Ring Stiffened Circular Hole. Dissertation for degree Ph.D. (Appl. Mech.), June 1974.
3. Pifko, A. and Goldberg, M. A.: Analysis of a Ring Reinforced Shallow Spherical Shell Subjected to an Eccentrically Located Concentrated Load. Presented at the University of New Brunswick, Fredericton, New Brunswick, Canada, on May 29, 1975. Also, Proc. of Fifth Congress of Applied Mechanics, Fredericton, New Brunswick, Canada; formerly POLY-AE/AM Report No. 74-5.
4. Yetman, William, Jr.: Dynamic Response of an Oval Ring. Thesis for the degree M. S. (Appl. Mech.), Polytechnic Institute of New York, June 1975.
5. Selsky, Gary M.: Free Vibration of an Oval Ring. Thesis for the degree M. S. (Appl. Mech.), Polytechnic Institute of New York, June 1975.
6. Chen, Y. N. and Yetman, W. R., Jr.: Dynamic Response of an Oval Ring. POLY-AE/AM Report No. 75-9, July 1975; AFOSR-TR-75-1534.
7. Chen, Y. N. and Kempner, Joseph: Modal Method for Free Vibration of Oval Cylindrical Shells with Simply Supported or Clamped Ends. POLY-AE/AM Report No. 75-14, August 1975; AFOSR-TR-76-1067.
8. Chen, Y. N. and Kempner, Joseph: Buckling of Oval Cylindrical Shells Under Compression and Asymmetric Bending. AIAA Journ., Vol. 14, No. 9, September 1976, pp. 1235-1240; formerly POLY-AE/AM Report No. 75-7.
9. Chen, Y. N. and Kempner, Joseph: Modal Method for Free Vibration of Finite Oval Cylindrical Shell with Free Ends. POLY-M/AE Report No. 76-1, September 1976; AFOSR-TR-76-1203.
10. Kempner, Joseph: Buckling and Vibrations of Noncircular Cylindrical Shells. Invited talk to be presented at the Engineering Section Meeting of the New York Academy of Sciences, April 13, 1977.



## APPENDIX

### Reports and Publications (1958-1976):

1. Patel, Sharad A. and Pandalai, K.A.V.: Torsion of Cylindrical and Prismatic Bars in the Presence of Primary Creep. PIBAL Report No. 417, April 1958. (AFOSR TN 58-303, AD 154 213).
2. Kempner, Joseph; Pandalai, K.A.V.; Patel, Sharad A. and French, Francis W.: Investigation of Plates and Shells under External Loading and Elevated Temperatures. PIBAL Report No. 481, Technical Summary Report for AFOSR Mechanics Branch-Contractor Meeting, October 1958.
3. Radok, J.R.M.: On Asymptotic Numerical Methods for Parabolic Equations. PIBAL Report No. 485, November 1958. (AFOSR TN 58-1072, AD 207 241).
4. Pandalai, K.A.V. and Patel, Sharad A.: Stress Distribution in Multi-Cellular Torque Boxes Due to Primary and Secondary Creep. PIBAL Report No. 480, December 1958. (AFOSR TN 58-1074, AD 207 243).
5. Patel, Sharad A. and Pandalai, K.A.V.: Stress Distribution in Beams of Thin-Walled Sections in the Presence of Creep. PIBAL Report No. 486, February 1959. (AFOSR TN 59-174, AD 211 314).
6. Pandalai, K.A.V. and Patel, Sharad A.: A Note on Shear Centers of Thin-Walled Closed Sections in the Presence of Creep. PIBAL Report No. 487, February 1959. (AFOSR TN 59-190, AD 211 311).
7. Radok, J.R.M. and Wang, K.: High Order Correct Difference Schemes for Multi-Dimensional Parabolic Equations. PIBAL Report No. 492, March 1959. (AFOSR TN 59-329, AD 213 676).
8. Patel, Sharad A. and Venkatraman, B.: Creep Behavior of Columns. PIBAL Report No. 422, May 1959. (AFOSR TN 59-530, AD 216 537).



9. French, Francis W. and Patel, Sharad A.: Creep Buckling of Cylindrical Shells Subjected to Uniform Axial Compression. PIBAL Report No. 489, May 1959. (AFOSR TN 59-538, AD 215 555).
10. French, Francis William, Jr.: Creep Buckling of Thin Circular Cylindrical Shells. Dissertation for the Degree of Doctor of Aeronautical Engineering, Polytechnic Institute of Brooklyn, June 1959.
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15. Sankaranarayanan, R.: A Note on the Impact Pressure Loading of a Rigid Plastic Spherical Shell. Reader's Forum, Jour. Aero./Space Sci., Vol. 28, No. 1, Jan. 1961, pp. 77-78; formerly PIBAL Rep. No. 564.
16. Lardner, Thomas J. and Pohle, Frederick V.: Application of Biot's Variational Principle in Heat Conduction. PIBAL Report No. 587, May 1961. (AFOSR 932).

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23. Patel, Sharad A. and Venkatraman, B.: Creep Bending of Compressible Plates. Int. Jour. Mech. Sci., Pergamon Press Ltd., Vol. 4, 1962, pp. 137-146; formerly PIBAL Rep. No. 590.
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35. Brady, Kevin J.: Supported Ring-Reinforced Cylindrical Shells under Concentrated Forces and Moments Applied to the Ring. Thesis for the Degree M.S. (Applied Mechanics), Polytechnic Institute of Brooklyn, June 1964.
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43. Kempner, Joseph: Postbuckling of Cylindrical Shells. Invited talk presented at Mechanical Engineering--Engineering Science Seminar, Rutgers University, May 10, 1965; based on PIBAL Rep. No. 694 and later results.
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56. Kempner, Joseph: Final Report for 1966. Investigation of Plates and Shells under External Loading and Elevated Temperatures. PIBAL Rep. No. 933, March 1967.
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58. Birnbaum, Michael Robert: Creep Stress Concentration at a Circular Hole in an Infinite Plate. Dissertation for the Degree Ph.D. (Applied Mechanics), Polytechnic Institute of Brooklyn, June 1967.
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